This supplement to your study for the Amateur Extra (Element 4) exam contains each question from the Extra Class (Element 4) Question Pool (ARRL, 2016) that requires mathematical formulas to answer. The references that were used in the development of this supplement are listed below.[1]

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Table of Metric Multiplication Factors

Multiplication Factor	Prefix	Symbol
1,000,000,000 = 10^9	giga	G
1,000,000 = 10^6	mega	M
1,000 = 10^3	kilo	k
100 = 10^2	hecto	h
1		
0.01 = 10^{-2}	centi	c
0.001 = 10^{-3}	milli	m
0.000001 = 10^{-6}	micro	µ
0.000000001 = 10^{-9}	nano	η
0.000000000001 = 10^{-12}	pico	p

References

ARRL. (2016). *The ARRL Extra Class License Manual, 11th Edition.* (M. Wilson K1RO, Ed.) Newington, CT: The National Association for Amateur Radio.

West, G. (2016). *2016 2020 Extra Class; FCC Element 4 Amateur Radio License Preparation, Seventh Edition.* (E. P. Nichols, Ed.) Niles, IL: Mater Publishing, Inc.

[1] Developed by Jamie Wright, W4ABE. If you have questions or find errors please contact me at (W4ABE@arrl.net)

Section E4: Amateur Radio Practices

E4B03: If a frequency counter with a specified accuracy of +/- 1.0 ppm reads 146,520,000 Hz, what is the most the actual frequency being measured could differ from the reading? (p. 7.11)

$$Error = \ f\ (Hz) \times \frac{Counter\ error}{1{,}000{,}000} = 146,520,000\ \times \frac{1}{1{,}000{,}000} = 146.52\ Hz$$

Where: f = frequency in Hertz

E4B04: If a frequency counter with a specified accuracy of +/- 0.1 ppm reads 146,520,000 Hz, what is the most the actual frequency being measured could differ from the reading? (p. 7.11)

$$Error = \ f\ (Hz) \times \frac{Counter\ error}{1{,}000{,}000} = 146,520,000\ \times \frac{0.1}{1{,}000{,}000} = 14.652\ Hz$$

Where: f = frequency in Hertz

E4B05: If a frequency counter with a specified accuracy of +/- 10 ppm reads 146,520,000 Hz, what is the most the actual frequency being measured could differ from the reading? (p. 7.10)

$$Error = \ f\ (Hz) \times \frac{Counter\ error}{1{,}000{,}000} = 146,520,000\ \times \frac{10}{1{,}000{,}000} = 1465.2\ Hz$$

Where: f = frequency in Hertz

E4B06: How much power is being absorbed by the load when a directional wattmeter connected between a transmitter and a termination load reads 100 W forward power and 25 W reflected power? (p. 9-36)

$$P_{LOAD} = P_{Forward} - P_{Reflected} = 100\ W - 25\ W = 75\ W$$

E4C06: What is the minimum discernable signal (MDS) for a receiver with a -174 dBm/Hz noise floor if a 400 Hz filter bandwidth is used? (p. 7-17)

Step 1 – Calculate the bandwidth ratio in dB = 10 log (400 Hz) = 26 dB
Step 2 – To get MDS, add that figure to -174 dBm = -174 dBm +26 dB = -148 dBm

E4C14: What transmit frequency might generate an image response signal in a receiver tuned to 14.300 MHz and which uses a 455 kHz frequency? (p. 7-19)

$$f_{image} = f_{tuned} + 2(f_{IF}) = 14.300 + 2(0.455) = 15.210\ MHz$$

where: f = frequency in megahertz

E4D05: What transmitter frequencies would cause an intermodulation-product signal in a receiver tuned to 146.70 MHz when a nearby station transmits on 146.52 MHz? (p. 7.22)

$$f_{IMD2} = 2f_1 - f_2 = 2(146.52) - 146.70 = 146.34\ MHz$$

$$f_{IMDx} = \frac{f_1 + f_2}{2} = \frac{146.52 + 146.70}{2} = 146.61\ MHz$$

where: *f* = frequency in megahertz

Section E5: Electrical Principles

E5A11: What is the half-power bandwidth of a parallel resonant circuit that has a resonant frequency of 7.1 Mhz and a Q of 150? (p. 4-33)

$$\Delta f = \frac{f_{resonant}}{Q} = \frac{7.1x10^6\ Hz}{150} = 4.733x10^4\ Hz = 47.3\ kHz$$

E5A12: What is the half-power bandwidth of a parallel resonant circuit that has a resonant frequency of 3.7 Mhz and a Q of 118? (p. 4-34)

$$\Delta f = \frac{f_r}{Q} = \frac{3.7x10^6\ Hz}{118} = 3.136x10^4\ Hz = 31.4\ kHz$$

E5A14: What is the resonant frequency of a series RLC circuit if R = 22 ohms, L is 50 microhenrys and C is 40 picofarads? (p. 4-29)

$$f_{resonant} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(50x10^6\ H)(40x10^{-12}F)}} = 3.56\ MHz$$

where: L = resistance in Henry's
C = capacitance in Farad's

E5A16: What is the resonant frequency of a parallel RLC circuit if R is 33 ohms, L is 50 microhenrys and C is 10 picofarads? (p. 4-30)

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(50x10^{-6}H)(10x10^{-12}F)}} = 7.12x10^6 = 7.12\ MHz$$

where: L = resistance in Henry's
C = capacitance in Farad's

E5B04: What is the time constant of a circuit having two 220-microfarad capacitors and two 1-Mohm resistors all in parallel? (p. 4-13)

$$R_\tau(parallel) = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{1 \times 1}{1+1} = 0.5\ M\Omega = 5x10^5\ \Omega$$

$$C_\tau(series) = C_1 + C_2 = 220 + 220 = 440\ \mu F = 440\ x\ 10^{-6}\ F$$

$$\tau = R \times C = (5x10^5\ \Omega) \times (440\ x\ 10^{-6}\ F) = 220\ seconds$$

Where: R_τ = time constant of resistors in ohms
C_τ = time constant of capacitors in Farads
τ = time constant

E5B07: What is the phase angle between voltage and current n a series RLC circuit if X_C is 500 Ω, R is 1 kΩ, and X_L is 250 Ω? (p. 4-24)

Note: X_C is greater than X_L, therefore, current leads the voltage/voltage lags the current.

Step 1 – add reactance and resistance together:

$$Z = 1000 + j250 - j500 = 1000 - j250\ \Omega$$

Step 2 – convert Z to a polar form:

$$\Theta = \tan^{-1}\frac{X}{R} = \tan^{-1}\frac{-250}{1000}$$

Step 3 – The phase angle (Θ) is equal to the angle of impedance. Θ = -14°

E5B08: What is the phase angle between voltage and current n a series RLC circuit if X_C is 100 Ω, R is 100 Ω, and X_L is 75 Ω? (p. 4-24)

Note: X_C is greater than X_L, therefore, current leads the voltage/voltage lags the current.

Step 1 – add reactance and resistance together:

$$Z = 100 + j75 - j100 = 100 - j25\ \Omega$$

Step 2 – convert Z to a polar form:

$$\Theta = \tan^{-1}\frac{X}{R} = \tan^{-1}\frac{-25}{100}$$

Step 3 – The phase angle is equal to the angle of impedance. Θ = -14°

E5B11: What is the phase angle between voltage and current n a series RLC circuit if X_C is 25 Ω, R is 100 Ω, and X_L is 50 Ω? (p. 4-24)

Note: X_C is less than X_L, therefore, voltage leads the current/current lags the voltage.

Step 1 – add reactance and resistance together:

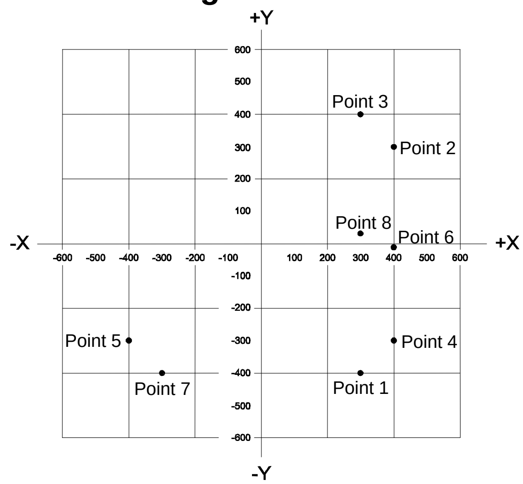
$$Z = 100 + j50 - j25 = 100 \mp 25\ \Omega$$

Step 2 – convert Z to a polar form:

$$\Theta = \tan^{-1}\frac{X}{R} = \tan^{-1}\frac{25}{100}$$

Step 3 – The phase angle is equal to the angle of impedance. Θ = 14°

Figure E5-2



E5C14: Using E5-2, which point represents the impedance of a circuit consisting of a 400 Ω resistor in series with a 38-pF capacitor at 14 MHz? (p.4-23)

Step 1 – Calculate the capacitor's reactance:

$$X_c = \frac{1}{2\pi f C} = \frac{1}{2\pi(14x10^6\ Hz)(38x10^{-12}\ F)} = -\ 300\ \Omega$$

where: f = frequency in Hertz
C = capacitance in Farad's

Step 2 – Use the rules to add the resistance and reactances together:

$$Z = 400 - j300\ \Omega$$

Step 3 – Locate the point on the graph, 400 units along the horizontal axis and -300 units on the vertical axis. Answer: Point 4

E5C15: Which point in Figure E5-2 best represents the impedance of a series circuit consisting of a 300 ohm resistor and an 18 microhenry inductor at 3.505 MHz? (p. 4-22)

Step 1 – Calculate the inductor's reactance:

$$X_L = 2\pi f L = 2\pi(3.505x10^6\ Hz)(18x10^{-6}\ H) = 396\ \Omega = 400\ \ \Omega$$

where: L = inductance in Henry's

Step 2 – Use Rule 1 to add the resistance and reactances together:

$$Z = 300 + j400\ \Omega$$

Step 3 – Locate the point on the graph, 300 units along the horizontal axis and 400 units on the vertical axis. Answer: Point 8

E5C16: Using E5-2, which point represents the impedance of a circuit consisting of a 300-Ω resistor in series with a 19-pF capacitor at 21.200 MHz? (p. 4-23)

Step 1 – Calculate the capacitor's reactance:

$$X_c = \frac{1}{2\pi f C} = \frac{1}{2\pi(21.2x10^6\ Hz)(19x10^{-12}\ F)} = -\ 400\ \Omega$$

where: *f* = frequency in Hertz
C = capacitance in Farad's

Step 2 – Use the rules to add the resistance and reactances together:

$$Z = 300 - j400\ \Omega$$

Step 3 – Locate the point on the graph, 300 units along the horizontal axis and -400 units on the vertical axis. Answer: Point 1

E5C17: Using E5-2, which point represents the impedance of a circuit consisting of a 300 Ω resistor in series with a 0.64-µH inductor and an 85-pF capacitor at 24,900 MHz? (p.4-24)

Step 1 – Calculate the capacitor's reactance:

$$X_c = \frac{1}{2\pi f C} = \frac{1}{2\pi(29400x10^3\ Hz)(85x10^{-12}\ F)} = -\ 75\ \Omega$$

where: *f* = frequency in Hertz
C = capacitance in Farad's

Step 2 – Calculate the inductor's reactance:

$$X_L = 2\pi f L = 2\pi(29400x10^3\ Hz)(0.64 \times 10^{-6}\ H) = 100\ \Omega$$

where: L = inductance in Henry's

Step 3 – Use the rules to add the resistance and reactances together:

$$Z = 300 + j - 75 + j100 = 300 + j25\ \Omega$$

Step 4 – Locate the point on the graph, 300 units along the horizontal axis and +25 units on the vertical axis. Answer: Point 8

E5D11: What is the power factor (PF) for an R-L circuit having a phase angle of 30°? (p. 4-27)

$$PF = \cos\theta = \cos 30 = 0.866$$

E5D12: How many watts are consumed in a circuit having a power factor of 0.2 if the input is 100-VAC at 4 amperes? (p. 4-27)

$$P_{APPARENT} = E \times I = 100 \times 4 = 400VA$$

$$P_{REAL} = P_{APPARENT} \times PF = 400VA \times 0.2 = 80W$$

where: I = current in amperes
E = voltage in volts

E5D13: How much power is consumed in a circuit consisting of a 100 Ω resistor in series with a 100-Ω inductive reactance and drawing 1 ampere of current? (p. 4-27)

Note: only resistance consumes power

$$P_{REAL} = I^2 \times R = 1A^2 \times 100\ \Omega = 100\ W$$

where: I = current in amperes
R = resistance in ohms

E5D15: What is the power factor for an R-L circuit having a phase angle of 45°? (p. 4-27)

$$PF = \cos\theta = \cos 45 = 0.707$$

E5D16: What is the power factor for an R-L circuit having a phase angle of 60°? (p. 4-27)

$$PF = \cos\theta = \cos 60 = 0.5$$

E5D17: How many watts are consumed in a circuit having a power factor of 0.6 if the input is 200 VAC at 5 amperes? (p. 4-28)

$$P_{APPARENT} = E \times I = 200 \times 5 = 1000VA$$

$$P_{REAL} = P_{APPARENT} \times PF = 1000VA \times 0.6 = 600W$$

where: I = current in amperes
E = voltage in volts

E5D18: How many watts are consumed in a circuit having a power factor of 0.71 if the input is 500 VAC? (p. 4-28)

$$P_{REAL} = P_{APPARENT} \times PF = 500VA \times 0.71 = 355W$$

Section E6: Circuit Components

E6D01: How many turns will be required to produce a 5-microhenry inductor using a powdered-iron toroidal core that has an inductance index value of 40 microhenries/100 turns? (p. 4-38)

$$N = 100\sqrt{\frac{L}{A_L}} = 100\sqrt{\frac{5}{40}} = 35.4 \; or \; 35 \; turns$$

where: A_L = Inductance Index in microHenry/100 turns
L = Inductance in microhenry

E6D11: How many turns will be required to produce a 1mH inductor using a core that has an inductance index value of 523/1000 turns? (p. 4-39)

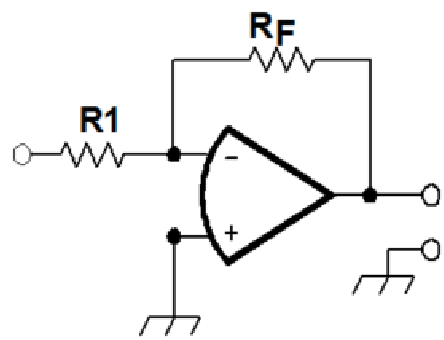
Note: for this question assume a ferrite core!

$$N = 1000\sqrt{\frac{L}{A_L}} = 1000\sqrt{\frac{1}{523}} = 43.7 \; or \; 43 \; turns$$

where: A_L = Inductance Index in microHenry/1000 turns
L = Inductance in microhenry

Section E7: Practical Circuits

Figure E7-4



E7G07: What magnitude of voltage gain can be expected from the circuit in Figure E7-4 when R1 is 10 ohms and RF is 470 ohms? (p. 6-9)

$$|A_V| = \frac{R_f}{R_1} = \frac{470\ \Omega}{10\ \Omega} = 47$$

E7G09: What will be the output voltage of the circuit shown in Figure E7-4 if R1 is 1000 ohms, RF is 10,000 ohms, and 0.23 volts DC is applied to the input? (p. 6-9)

$$|A_V| = \frac{R_f}{R_1} = \frac{10000\ \Omega}{1000\ \Omega} = 10 = -10\ (since\ circuit\ is\ inverting)$$

$$V_{out} = |A_V| \times V_{in} = -10 \times 0.23 = -2.3\ V$$

E7G10: What absolute voltage gain can be expected from the circuit in Figure E7-4 when R1 is 1800 ohms and RF is 68 kilohms? (p. 6-9)

$$|A_V| = \frac{R_f}{R_1} = \frac{68x10^3\ \Omega}{1800\ \Omega} = 37.8\ V = 38\ V$$

E7G11: What absolute voltage gain can be expected from the circuit in Figure E7-4 when R1 is 3300 ohms and RF is 47 kilohms? (p. 6-9)

$$|A_V| = \frac{R_f}{R_1} = \frac{47x10^3\ \Omega}{3300\ \Omega} = 14\ V$$

Section E8: Signals and Emissions

E8B03: What is the modulation index of an FM-phone signal having a maximum frequency deviation of 3000 Hz either side of the carrier frequency when the modulating frequency is 1000 Hz? (p. 8-4)

$$Modulation\ Index = \frac{D_{MAX}}{m} = \frac{3000\ Hz}{1000\ Hz} = 3$$

where: D_{MAX} = peak deviation in Hertz
m = modulating frequency in hertz at any given instant

E8B04: What is the modulation index of an FM-phone signal having a maximum carrier deviation of plus or minus 6 kHz when modulated with a 2 kHz modulating frequency? (p. 8-4)

$$Modulation\ Index = \frac{D_{MAX}}{m} = \frac{6000\ Hz}{2000\ Hz} = 3$$

where: D_{MAX} = peak deviation in Hertz
m = modulating frequency in hertz at any given instant

E8B05: What is the deviation ratio of an FM-phone signal having a maximum frequency swing of plus-or-minus 5 kHz when the maximum modulation frequency is 3 kHz? (p. 8-4)

$$Deviation\ Ratio = \frac{D_{MAX}}{M} = \frac{5000\ Hz}{3000\ Hz} = 1.67$$

where: D_{MAX} = peak deviation in Hertz
M = maximum modulating frequency in hertz

E8B06: What is the deviation ratio of an FM-phone signal having a maximum frequency swing of plus or minus 7.5 kHz when the maximum modulation frequency is 3.5 kHz? (p. 8-4)

$$Deviation\ Ratio = \frac{D_{MAX}}{M} = \frac{7500\ Hz}{3500\ Hz} = 2.14$$

where: D_{MAX} = peak deviation in Hertz
M = maximum modulating frequency in hertz

E8C06: What is the necessary bandwidth of a 170-hertz shift, 300-baud ASCII transmission? (p. 8-11)

$$BW = (K \times Shift) + B = (1.2 \times 170\ Hz) + 300 = 504\ Hz = 0.5\ kHz$$

where: BW = bandwidth in Hz
K = 1.2 (constant)
Shift = frequency shift in Hz
B = symbol rate in baud

E8C07: What is the necessary bandwidth of a 4800-Hz frequency shift, 9600-baud ASCII FM transmission? (p. 8-11)

$$BW = (K \times Shift) + B = (1.2 \times 4800\ Hz) + 9600 = 15360\ Hz = 15.3\ kHz$$

where: BW = bandwidth in Hz
K = 1.2 (constant)
Shift = frequency shift in Hz
B = symbol rate in baud

Section E9: Antennas

E9A12: How much gain does an antenna have compared to a 1/2-wavelength dipole when it has 6 dB gain over an isotropic antenna? (p. 9-4)

$$Gain\ in\ dBd = Gain\ in\ dBi - 2.15 = 6 - 2.15 = 3.85\ dBd$$

where: dBi = antenna gain compared to an isotropic radiator
dBd = antenna gain compared to a reference dipole
2.15 is a constant (diploe has 2.15 dB gain over an isotropic radiator)

E9A13: How much gain does an antenna have compared to a 1/2-wavelength dipole when it has 12 dB gain over an isotropic antenna? (p. 9-4)

$$Gain\ in\ dBd = Gain\ in\ dBi - 2.15 = 12 - 2.15 = 9.85\ dBd$$

where: dBi = antenna gain compared to an isotropic radiator
dBd = antenna gain compared to a reference dipole
2.15 is a constant (diploe has 2.15 dB gain over an isotropic radiator)

E9A15: What is the effective radiated power relative to a dipole of a repeater station with 150 watts transmitter power output, 2 dB feed line loss, 2.2 dB duplexer loss, and 7 dBd antenna gain? (p. 9-28)

$$System\ gain = sum\ of\ all\ the\ losses\ and\ gains = -2dB - 2.2\ dB + 7\ dB = 2.8\ dB$$

$$ERP = P \times 10^{\frac{system\ gain}{10}} = P \times log^{-1}\left(\frac{system\ gain}{10}\right) = 150 \times 10^{0.28} = 286\ W$$

where: ERP = effective radiated power
P = power output

E9A16: What is the effective radiated power relative to a dipole of a repeater station with 200 watts transmitter power output, 4 dB feed line loss, 3.2 dB duplexer loss, 0.8 dB circulator loss, and 10 dBd antenna gain? (p. 9-28)

$$System\ gain = -4dB - 3.2\ dB - 0.8 + 10\ dB = 2\ dB$$

$$ERP = P \times 10^{\frac{system\ gain}{10}} = P \times log^{-1}\left(\frac{system\ gain}{10}\right) = 200 \times log^{-1}(\frac{2}{10}) = 317\ W$$

where: ERP = effective radiated power
P = power output

E9A17: What is the effective radiated power of a repeater station with 200 watts transmitter power output, 2 dB feed line loss, 2.8 dB duplexer loss, 1.2 dB circulator loss, and 7 dBi antenna gain? (p. 9-28)

$$System\ gain = -2dB - 2.8\ dB - 1.2 + 7\ dB = 1\ dB$$

$$ERP = P \times 10^{\frac{system\ gain}{10}} = P \times log^{-1}\left(\frac{system\ gain}{10}\right) = 200 \times log^{-1}(\frac{1}{10}) = 252\ W$$

where: ERP = effective radiated power
P = power output

E9F05: What is the approximate physical length of a solid polyethylene dielectric coaxial transmission line that is electrically one-quarter wavelength long at 14.1 MHz? (p. 9-33)

$$Length = VF \times \frac{300}{f} = 0.66 \times \frac{300}{14.1} = 14.1\ m$$

$$Length\ for \frac{1}{4}\lambda = 0.25 \times Length = 0.25 \times 14.1 = 3.5\ m$$

where: VF = Velocity Factor for solid polyethylene dielectric coaxial transmission, 0.66
f = frequency in MHz

E9F06: What is the approximate physical length of an air-insulated, parallel conductor transmission line that is electrically one-half wavelength long at 14.10 MHz? (p. 9-33)

$$Length = VF \times \frac{300}{f} = 0.95 \times \frac{300}{14.1} = 20.2\ m$$

$$Length\ for \frac{1}{2}\lambda = 0.5 \times Length = 0.5 \times 20.2 = 10\ m$$

where: VF = Velocity Factor for air-insulated, parallel conductor transmission line, 0.95
f = frequency in MHz

E9F09: What is the approximate physical length of a solid polyethylene dielectric coaxial transmission line that is electrically one-quarter wavelength long at 7.2 MHz?

$$Length = VF \times \frac{300}{f} = 0.66 \times \frac{300}{7.2} = 27.5\ m$$

$$Length\ for \frac{1}{4}\lambda = 0.25 \times Length = 0.25 \times 27.5 = 6.9\ m$$

where: VF = Velocity Factor for solid polyethylene dielectric coaxial transmission, 0.66
f = frequency in MHz